International Journal of Astrophysics and Space Science
2015; 3(3): 50-54
Published online May 13, 2015 (http://www.sciencepublishinggroup.com/j/ijass)
doi: 10.11648/j.ijass.20150303.14
ISSN: 2376-7014 (Print); ISSN: 2376-7022 (Online)

Dynamics of a Charged Scalar Field Thin Shell

A. Eid[1, 2]

[1]Department of Physics, College of Science, Al Imam Mohammad Ibn Saud Islamic University (IMSIU), Riyadh, KSA
[2]Department of Astronomy, Faculty of Science, Cairo University, Giza, Egypt

Email address:
aeid06@yahoo.com

To cite this article:
A. Eid. Dynamics of a Charged Scalar Field Thin Shell. *International Journal of Astrophysics and Space Science*.
Vol. 3, No. 3, 2015, pp. 50-54. doi: 10.11648/j.ijass.20150303.14

Abstract: This paper deals with the motion of a charged spherically symmetric thin shell composed of a scalar field. The equations of motion resulting from the matching conditions of the two surrounding spacetimes which are described by the Reissner Nordstrom metric are derived. I evaluate the case of massless scalar field, where the scalar field potential is zero, also I evaluate the case of massive scalar field.

Keywords: General Relativity, Scalar Shell, Cosmology, Gravitation, Astrophysics

1. Introduction

The study of the dynamics of a shell separating two background in the context of general relativity has been developed in a powerful and direct formalism since the pioneer work of Israel [1] and applied to the charged shell by Kuchar [2]. It has been applied to cosmology, mainly to inflation [3], and to modeling the dynamics of the border between two regions in different states, like bubbles or between two given spaces [4]. Nunez and de Oliveira [5] used it in the study of the dynamics of massive shell ejected in supernova explosion. Israel [1] found a set of invariant boundary conditions connecting the relation between the extrinsic curvature of a shell on its both sides and the matter of this shell.

The cornerstone of these studies consists of analyzing the behavior of the intrinsic and extrinsic curvatures across the shell. It turns out that the intrinsic curvature changes smoothly, whereas the extrinsic curvature suffers a jump determined by the energy-momentum tensor of the shell. This jump has to be in accordance with the corresponding boundary conditions which, in turn, can be obtained from the Einstein field equations of the surrounding spacetime, and from the equations which govern the behavior of the matter in the shell.

In the relativistic astrophysics, the thin shell equations help to study the properties of the compact objects, boson stars are such compact objects that are composed of scalar field [6,7]. More recently, boson stars have also been proposed as a candidate for composing dark matter [8].

The paper is organized as follows. In Section 2 I briefly review the Darmois – Israel formalism. In Section 3 the equations of motion of charged thin shell are derived. The equations of motion of charged scalar field shell in the case of massless and massive scalar field with arbitrary scalar potential are presented in section 4. A general conclusion is given in section 5.

2. The Darmois Israel Formalism

Consider two distinct spacetime manifolds M_+ and M_- with metrics given by $g^{+}_{\mu\nu}(x^{\mu}_{+})$ and $S^{ij}\bar{K}_{ij} = \left[-T_{\mu\nu}n^{\mu}n^{\nu} - \frac{\Lambda}{8\pi}\right]^{+}_{-}$, in terms of independently defined coordinate systems $x^{\mu}_{\pm}$. The manifolds are bounded by hypersurfaces Σ_+ and Σ_-, respectively, with induced metrics $g^{\pm}_{ij}$. The hypersurfaces are isometric, i.e. $g^{+}_{ij}(\xi) = g^{-}_{ij}(\xi) = g_{ij}(\xi)$, in terms of the intrinsic coordinates, invariant under the isometry. A single manifold M is obtained by gluing together M_+ and M_- at their boundaries, i.e. $M = M_+ \cup M_-$, with the natural identification of the boundaries $\Sigma = \Sigma_+ = \Sigma_-$. The second fundamental forms (extrinsic curvature) associated with the two sides of the shell are:

$$K^{\pm}_{ij} = -n^{\pm}_{\gamma}\left(\frac{\partial^2 x^{\gamma}}{\partial\xi^i\partial\xi^j} + \Gamma^{\gamma}_{\alpha\beta}\frac{\partial x^{\alpha}}{\partial\xi^i}\frac{\partial x^{\beta}}{\partial\xi^j}\right)\Big|_{\Sigma} \quad (1)$$

where $n_\gamma^\pm$ are the unit normal 4-vector to Σ in M, with $n_\mu n^\mu = 1$ and $n_\mu e_i^\mu = 0$. The Israel formalism requires that the normal point from M_- to M_+. For the case of a thin shell K_{ij} is not continuous across Σ, so that, the discontinuity in the second fundamental form is defined as $\left[K_{ij}\right] = K_{ij}^+ - K_{ij}^-$. The Einstein equation determines the relations between the extrinsic curvature and the three dimensional intrinsic energy momentum tensor are given by the Lanczos equations,

$$S_{ij} = \frac{-1}{8\pi}\left(\left[K_{ij}\right] - [K] g_{ij}\right) \tag{2}$$

where $[K]$ is the trace of $\left[K_{ij}\right]$ and S_{ij} is the surface stress-energy tensor on Σ. The first contracted Gauss-Kodazzi equation or the "Hamiltonian" constraint

$$G_{\mu\nu} n^\mu n^\nu = \frac{1}{2}(K^2 - K_{ij}K^{ij} - {}^3R) \tag{3}$$

with the Einstein equations provide the evolution identity

$$S^{ij}\bar{K}_{ij} = \left[-T_{\mu\nu} n^\mu n^\nu - \frac{\Lambda}{8\pi}\right]_-^+ \tag{4}$$

The convention $[X] = X^+ - X^-$, and $\bar{X} = \frac{1}{2}(X^+ + X^-)$, is used. The second contracted Gauss- Kodazzi equation or the "ADM" constraint,

$$G_{\mu\nu} e_i^\mu n^\nu = K_{i;j}^j - K_{,i} \tag{5}$$

With the Lanczos equations gives the conservation identity

$$S_{j;i}^i = \left[T_{\mu\nu} e_i^\mu n^\nu\right]_-^+ \tag{6}$$

If the shell is composed of a perfect fluid, the energy momentum tensor is

$$S^{ij} = (\sigma + p)u^i u^j + p\gamma^{ij} \tag{7}$$

where σ and p are the surface energy density and surface pressure of the matter on the shell, respectively and $u^i = d\xi^i / d\tau$ is the three-velocity of that matter which moves perpendicular to n^μ. One may obtain an equation governing the behavior of the radial pressure in terms of the surface stresses at the junction boundary from the following identity:

$$\left[T_{\mu\nu} n^\mu n^\nu\right] = \frac{1}{2}(K_j^{i+} + K_j^{i-})S_j^i \tag{8}$$

For spherically symmetric thin shell, the Lanczos equations reduce to

$$\sigma = \frac{-1}{4\pi}\left[K_\theta^\theta\right] \tag{9}$$

$$p = \frac{1}{8\pi}\left(\left[K_\tau^\tau\right] + \left[K_\theta^\theta\right]\right) \tag{10}$$

If the surface stress-energy terms are zero, the junction is denoted as a boundary surface. If surface stress terms are present, the junction is called a thin shell.

3. Equations of Motion of Charged Thin Shell

The interior and exterior space-times are described by the Reissner-Nordstrom (RN) metrics given by:

$$ds_\pm^2 = -f_\pm dt^2 + f_\pm^{-1} dr^2 + r^2 d\Omega^2 \tag{11}$$

with

$$d\Omega^2 = d\theta^2 + \sin^2\theta d\varphi^2$$

is the line element on the unit sphere, and

$$f_\pm = 1 - \frac{2m_\pm}{r} + \frac{q_\pm^2}{r^2} \tag{12}$$

where $m_\pm$ and $q_\pm$ are the mass and charge, respectively. The suffix '+' denotes a quantity evaluated just outside the shell and '-' just inside the shell.

Let r be the area radius, i.e. the radial coordinate such that $A = 4\pi r^2$ is the area of the spheres of symmetry at constant r. The area radius is continuous across Σ, which is not true for the time coordinates. Let the equation of the shell be $r_\pm = R_\pm(\tau)$, the history of the shell is described by the hypersurface $x_\pm^\alpha = x_\pm^\alpha(\tau, \theta, \varphi)$, in the regions $M^\pm$, respectively; the function $R(\tau)$ is the proper radius of Σ, describes the time evolution of the shell. The induced intrinsic metric on Σ is written as

$$ds^2 = -d\tau^2 + R^2(\tau)(d\theta^2 + \sin^2\theta d\varphi^2) \tag{13}$$

where τ is the proper time of the shell. The four velocity is given by

$$U_\pm^\mu = (f_\pm^{-1}\sqrt{f_\pm + \dot{R}^2}, \dot{R}, 0, 0)$$

where the overdot denotes a derivative with respect to τ. The unit normal to the junction surface is $n_\pm^\mu = (\dot{R} f_\pm^{-1}, \sqrt{f_\pm + \dot{R}^2}, 0, 0)$. Using equation (1), the non-trivial components of the extrinsic curvature are given by:

$$K_\theta^{\theta\pm} = K_\varphi^{\varphi\pm} = \frac{1}{R}\sqrt{f_\pm + \dot{R}^2} \tag{14}$$

$$K_\tau^{\tau\pm} = \frac{1}{\sqrt{f_\pm + \dot{R}^2}}\left(\frac{m_\pm}{R^2} - \frac{q^2}{R^3} + \ddot{R}\right)$$

Assume that charge in both regions is the same,

$q_+ = q_- = q$.

Therefore, the Lanczos equations (9,10), with the extrinsic curvature equations (14), are given by

$$\sigma = \frac{-1}{4\pi R}\left[\sqrt{f+\dot{R}^2}\right]_-^+ \qquad (15)$$

$$p = \frac{1}{8\pi R}\left[\frac{1-\frac{m}{R}+\frac{2q^2}{R^2}+\dot{R}^2+R\ddot{R}}{\sqrt{f+\dot{R}^2}}\right]_-^+ \qquad (16)$$

Rearranging equation (15) into the form

$$4\pi\sigma R = \sqrt{f_- + \dot{R}^2} - \sqrt{f_+ + \dot{R}^2} \equiv \frac{M}{R} \qquad (17)$$

where $M = 4\pi\sigma R^2$ is the rest mass of the shell. Equation (17) can be written in the form

$$\dot{R}^2 = \frac{M^2}{4R^2} + \frac{R^2}{4M^2}(f_- - f_+)^2 - \frac{1}{2}(f_- + f_+) \qquad (18)$$

It represents the energy equation of the shell, and can be called the expansion law of the shell. Equation (18) can be written in the following dynamical form

$$\dot{R}^2 + V_{eff}(R) = 0 \qquad (19)$$

where

$$V_{eff}(R) = 1 - \frac{M^2}{4R^2} - \left(\frac{m_+ - m_-}{M}\right)^2 - \frac{2\overline{m}}{R} + \frac{q^2}{R^2} \qquad (20)$$

with, $\overline{m} = \frac{1}{2}(m_+ + m_-)$ is the effective potential that determines the motion of the shell. Equation (19) represent the energy conservation law which states that the sum of the "kinetic component" ($\dot{R}^2$) and "potential component" (Veff) equals zero at any time. This equation has a form similar to Keplerian equations with total zero energy and zero charge. The solutions allowed from equation (19) are only those for which the effective potential is negative or zero. The case of (Veff = 0) correspond either to a static configuration or to the turning points, i.e., orbits of extremely radius R.

4. Equations of Motion of Charged Scalar Field Shell

By using the transformation ($u_i = \phi_{,i}\big/\sqrt{-\phi_{,j}\phi^{,j}}$) [2], the surface energy density and pressure of a perfect fluid can be written in terms of the potential V(ϕ) of a scalar field. Therefore,

$$\sigma = \frac{-1}{2}\left[\phi_{,i}\phi^{,i} - 2V(\phi)\right], \qquad (21)$$

$$p = \frac{-1}{2}\left[\phi_{,i}\phi^{,i} + 2V(\phi)\right],$$

From (7) the energy momentum tensor of the scalar field will be:

$$S^{ij} = \nabla^i\phi\nabla^j\phi - \gamma^{ij}\left[\frac{1}{2}(\nabla\phi)^2 + V(\phi)\right]$$

Since, ϕ depends only on τ, then equation (21) leads to

$$\sigma = \frac{1}{2}\left[\dot{\phi}^2 + 2V(\phi)\right], \qquad (22)$$

$$p = \frac{1}{2}\left[\dot{\phi}^2 - 2V(\phi)\right],$$

The total mass of the shell (M= σA), in terms of the scalar field, is

$$M = 2\pi R^2\left[\dot{\phi}^2 + 2V(\phi)\right] \qquad (23)$$

Using equations (14), (22) and (23) to get

$$\ddot{\phi} + \frac{2\dot{R}}{R}\dot{\phi} + \frac{\partial V}{\partial\phi} = 0 \qquad (24)$$

which represent the Klein-Gordon equation, $\Box\phi + \partial V/\partial\phi = 0$, in the particular coordinate system of the shell. The full dynamics of the charged scalar shell and the scalar field, will be determined by the Klein-Gordon equation (24), and by the equation of energy conservation (19) (with the effective potential (20)) simultaneously for $\phi(\tau)$ and $R(\tau)$. The effective potential, in terms of the scalar field, is:

$$V_{eff}(R) = 1 - \left(\frac{m_+ - m_-}{2\pi R^2(\dot{\phi}^2 + 2V)}\right)^2 - \frac{2\overline{m}}{R} + \frac{q^2}{R^2} - \pi^2 R^2(\dot{\phi}^2 + 2V)^2 \qquad (25)$$

The exact solution of the KG equation and the equation of motion (19) with (25) is not possible. Taking into account the transparency condition, $\left[G_{\mu\nu}U^{\mu}n^{\nu}\right]_-^+ = 0$, the conservation identity, equation (6), provides the simple relationship:

$$\frac{d}{d\tau}(\sigma A) + p\frac{dA}{d\tau} = 0$$

where $A = 4\pi R^2$ is the area of the shell and $M \equiv \sigma A$ is the total mass associated to the shell . It can be written in the form

$$\dot{\sigma} + \frac{2\dot{R}}{R}(p+\sigma) = 0 \qquad (26)$$

and

$$R\sigma_{,R} + 2(p+\sigma) = 0 \qquad (27)$$

when $\dot{R} \neq 0$. There are two strategies to integrate this equation. The first one deals with $p = p(R)$, and the second strategy deals with an equation of state $p = p(\sigma)$. The first approach leads to the solution

$$\sigma(R) = \frac{-2}{R^2} \int_{R_\circ}^{R} p(\bar{R})\bar{R}d\bar{R}$$

The second approach leads to the solution

$$\int_{\sigma_\circ}^{\sigma} \frac{d\sigma}{p(\sigma)+\sigma} = \ln(\frac{c}{R^2}) \tag{28}$$

where c is a constant of integration. In the present case, this equation can be solved by using the equation of state of polytrope-type:

$$p = \kappa\sigma^{\gamma} \tag{29}$$

where κ, $\gamma = (1+n^{-1})$ and n are constants. Then, from equation (28) the scalar density is written in the form

$$\sigma = \left(\frac{c^{\gamma-1}}{R^{2(\gamma-1)} - kc^{\gamma-1}}\right)^{1/\gamma-1} = \left(\frac{c^{1/n}}{R^{2/n} - kc^{1/n}}\right)^{n}$$

So that the scalar field and the scalar potential are given by

$$\dot{\phi}^2 = \sigma + p = \frac{cR^{2(\gamma-1)}}{[R^{2(\gamma-1)} - kc^{\gamma-1}]^{\gamma/\gamma-1}} = \frac{cR^{2/n}}{[R^{2/n} - kc^{1/n}]^{n+1}}$$

$$2V(\phi) = \sigma - p = \frac{c(R^{2(\gamma-1)} - 2kc^{\gamma-1})}{[R^{2(\gamma-1)} - kc^{\gamma-1}]^{\gamma/\gamma-1}} = \frac{c(R^{2/n} - 2kc^{1/n})}{[R^{2/n} - kc^{1/n}]^{n+1}}$$

For γ = k =1, equation (29) will be p = σ, then (28) become $\sigma = c^2R^{-4}$.

4.1. Massless Scalar Field

When the scalar potential field V(ϕ) is zero, a scalar field becomes massless and the Klein-Gordon equation (24) can be integrated to get, $\dot{\phi} = c/R^2$. Thus, the equation of motion (19) and (25) can be written in the form:

$$\dot{R}^2 + 1 - \left(\frac{m_+ - m_-}{4\pi c^2}\right)^2 R^4 - \frac{2\bar{m}}{R} + \frac{q^2}{R^2} - \frac{\pi^2 c^4}{R^6} = 0 \tag{30}$$

The massless scalar shell may expand or collapse depending on the sign of velocity ($\dot{R}$) of the shell with respect to stationary observer. The effective potential tends to negative infinity when R tends to infinity, and then the shell expands to infinity or collapses to zero. Equation (19) and (30) become

$$\dot{R}^2 + 1 - aR^4 - bR^{-1} + q^2R^{-2} - c_1R^{-6} = 0$$

where $a = \left(\frac{m_+ - m_-}{2\pi c^2}\right)^2$, $b = 2\bar{m}$ and $c_1 = \pi^2 c^4$. For the turning points ($\dot{R} = 0$), the algebraic equation for R(τ) will be

$$aR^{10} - R^6 + bR^5 - q^2R^4 + c_1 = 0$$

The shell will expand with the initial value of R to maximum radius and collapses into the central Schwarzschild mass.

4.2. Massive Scalar Field

For a massive scalar field: $V(\phi) = m^2\phi^2$. From (22), the surface density and pressure will be

$$\dot{\phi}^2 = \sigma + p \tag{31}$$

$$2V = 2m^2\phi^2 = \sigma - p$$

I will discuss the motion of a massive scalar field in two approach of p by taking p as an explicit function of R, and as an explicit function of σ (polytropic type).

In the first approach p = p(R), with $p = p_\circ e^{-\zeta R}$, where $p_\circ$ and ζ are constants. From (27) the surface density is

$$\sigma = \frac{\xi}{R^2} + \frac{2p_\circ}{R^2} e^{-\zeta R} (\frac{1+\zeta R}{\zeta^2})$$

with ξ is the constant of integration. Thus, ϕ and $\dot{\phi}$ are obtained from equation (31):

$$V(\phi) = m^2\phi^2 = \frac{\xi}{2R^2} - \frac{p_\circ}{2} e^{-\zeta R} (1 - \frac{2(1+\zeta R)}{\zeta^2 R^2}) \tag{32}$$

$$\dot{\phi}^2 = \frac{\xi}{R^2} + p_\circ e^{-\zeta R} (1 + \frac{2(1+\zeta R)}{\zeta^2 R^2})$$

These equations satisfy the KG equation (24). From (32) and (25) the effective potential become

$$V_{eff}(R) = 1 - \left(\frac{\tilde{m}_+ - \tilde{m}_-}{\tilde{M}}\right)^2 - \frac{M_*}{R_*}(\frac{2\tilde{\bar{m}}}{\tilde{R}}) + \frac{\tilde{q}^2}{\tilde{R}^2}(\frac{q_*}{R_*})^2 - (\frac{\tilde{M}M_*}{2\tilde{R}R_*})^2 \tag{33}$$

with, $\tilde{\bar{m}} = \frac{1}{2}(\tilde{m}_+ + \tilde{m}_-)$, where the masses $\tilde{m}_+, \tilde{m}_-, \tilde{M}$ and the radius $\tilde{R}$ are dimensionless, and $\tilde{M}$ has the explicit form:

$$\tilde{M} = \frac{4\pi\xi}{M_*} + \frac{8\pi p_\circ}{\zeta^2 M_*}(1 + \zeta\tilde{R}R_*)e^{-\zeta\tilde{R}R_*}$$

From (19), the behavior of the effective potential (33) with $\dot{R} = 0$ implies that, the shell stops with two points and recoils. There exist a different values of charge parameter for which the scalar field shell executes an oscillatory motion. This oscillation occur at two points where Veff cuts the horizon.

In the second approach, $p = p(\sigma)$, a polytropic type equation of state (29) with $\gamma = 3/2$ and $k = c = 1$, will be $p = \sigma^{3/2}$. From (28) the density become

$$\sigma = (\frac{R^2}{c - R^2})^2$$

Thus ϕ and $\dot{\phi}$ will be

$$\dot{\phi}^2 = \frac{cR^4}{(c - R^2)^3}$$

$$2V(\phi) = \frac{R^4(c - 2R^2)}{(c - R^2)^3}$$

The effective potential (25) become

$$V_{eff}(R) = 1 - \left(\frac{\tilde{m}_+ - \tilde{m}_-}{\tilde{M}}\right)^2 - \frac{M_*}{R_*} \tag{34}$$
$$(\frac{\tilde{m}_+ + \tilde{m}_-}{\tilde{R}}) + \frac{\tilde{q}^2}{\tilde{R}^2}(\frac{q_*}{R_*})^2 - (\frac{\tilde{M}M_*}{2\tilde{R}R_*})^2$$

with

$$\tilde{M} = \frac{4\pi R_*^2 \tilde{R}^2}{M_*}(\frac{\tilde{R}^2 R_*^2}{c - R_*^2 \tilde{R}^2})^2$$

Equation (19) and (34) become

$$\dot{R}^2 - 4\pi^2 R^{22} + R^{20} - 2\overline{m}R^{19} + lR^{18} + 8c\overline{m}R^{17} + zR^{16}$$
$$-12c^2\overline{m}R^{15} + hR^{14} + 8c^3\overline{m}R^{13} + dR^{12} - 2c^4\overline{m}R^{11} + gR^{10}$$
$$-70c^4wR^8 + 56c^5wR^6 - 28c^6wR^4 + 8c^7wR^2 - wc^8 = 0$$

where $w = \left(\frac{m_+ - m_-}{2\pi}\right)^2$, $z = 6c^2 - 4cq^2 - w$ and $h = -4c^3 + 6c^2q^2 + 8cw$, $d = c^4 - 4c^3q^2 - 28c^2w$, $g = c^4q^2 + 56c^3w$, $l = q^2 - 4c$

Moreover, for the turning points, $\dot{R} = 0$, the algebraic equation of R(τ) will be:

$$-4\pi^2 R^{22} + R^{20} - 2\overline{m}R^{19} + lR^{18} + 8c\overline{m}R^{17} + zR^{16}$$
$$-12c^2\overline{m}R^{15} + hR^{14} + 8c^3\overline{m}R^{13}$$
$$+dR^{12} - 2c^4\overline{m}R^{11} + gR^{10} - 70c^4wR^8 + 56c^5wR^6$$
$$-28c^6wR^4 + 8c^7wR^2 - wc^8 = 0$$

During the expansion stage the scalar field is decreased due to the term $2\dot{R}\dot{\phi}/R$ in the Klein-Gordon equation (24); the shell starts collapsing, $\dot{R}$ changes its sign, and thus the amplitude of ϕ is increased until the shell completes the collapse.

5. Conclusion

In the framework of Darmois-Israel formalism, the equations of motion of a charged spherically symmetric thin shell have been formulated by taking the internal and external regions to the boundary surface as RN solution. The equations of motion are originally derived for perfect fluid in polytropic equation of state and then are written in terms of scalar field. The complete dynamical behavior of the charged scalar field thin shell is described by the equation of motion (19) and the KG equation (24).

There exist two different approaches of P to integrate these equations of motion: in the first one, by taking the pressure of the shell as an explicit function of radius R, and in the second one by taking the pressure as an explicit function of the energy-density σ of the shell (polytropic type). Both strategies are then used to analyze the cases of a massless as well as a massive scalar shell. In the both (massless and massive scalar field) cases, the three possible phases (expanding, collapsing and oscillating) during the dynamics of the scalar field in the present configuration are exist.

There are a difference between the model presented here and the model used in [9], where the gravitational field outside the shell is described by the Vaidya metric, also with the model used in [10], where the gravitational field outside the shell is described by the Schwarzschild metric.

References

[1] W. Israel, Nuovo Cimento B, 44, 1, 1966.

[2] K. Kuchar, Czech J. Phys. B, 18, 435,1968.

[3] V.A. Berezin, V.A. Kuzmin and I.I. tkachev, Phys. Rev. D., 36, 2919, 1987.

[4] H. Sato, Prog. Theor. Phys., 80, 96, 1988.

[5] D. Nunez and H.P. de Oliveira, Phys. Lett. A., 214, 227, 1996.

[6] D. J. Kaup, Phys. Rev., 172, 1331, 1968.

[7] R. Ruffini and S. Bonazzola, Phys. Rev., 187, 1767, 1969.

[8] P. Jetzer, Phys. Rep., 220, 163, 1992.

[9] T. Koike and T. Mishima, Phys. Rev. D., 51, 4045, 1995.

[10] D. Nunez, H. Quevedo and M. Salgado, Phys. Rev. D., 58, 083506, 1998.