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# The Effect of Thermal Parameters on the Flow Temperature of a Magnetized Plasma in a Sphere

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**Abstract:** The effect of thermal parameters on the flow temperature of a magnetized plasma in a sphere was studied. The study models astrophysical environments such as the Sun, which have spherical outline. The governing equations of the problem were obtained based on the Navier-Stokes equations under the Boussinesq's approximation. The solutions to the resulting equations were sought by means of the general perturbation method and the results were graphically represented with radial distance,  $r = 1.0$  on the figures corresponding to the surface of the sphere. The thermal parameters; particularly, the radiation parameter,  $N^2$  and free convection parameter,  $Gr$ . were investigated in this study with a view to determine the effect of varying these parameters on the plasma flow temperature. Increasing both  $N^2$  and  $Gr$ . led to a decrease in the plasma flow temperature in the sphere. However, above the sphere (i.e. at radial distances,  $r > 1.0$ ) where the plasma density is sparse, increasing  $N^2$  and  $Gr$ . produced a corresponding increase in the plasma flow temperature. The decrease in the plasma flow temperature within the sphere with increase in the thermal parameters was observed to be more significant between radial distances,  $r = 0.25$  and  $r = 0.7$  (i.e.,  $0.25 \leq r \leq 0.7$ ) than between  $r = 0.7$  and  $r = 1.0$  (i.e.,  $0.7 \leq r \leq 1.0$ ). This is attributable to the prevalence of partially ionized heavy elements within  $0.7 \leq r \leq 1.0$  (corresponding to the convection zone of the solar interior) which trap the high energy photons thereby reducing the rate of radiative heat loss.

**Keywords:** Thermal Parameters, Flow Temperature, Magnetized Plasma

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## 1. Introduction

The Sun is powered with the ability to replenish the huge amount of energy lost to solar space from the nuclear fusion process at the core. Of the very high temperature of about  $1.5 \times 10^7$  K at the core of the Sun, the temperature at the solar surface is only about 5,780 K. It therefore means that most of the energy is lost on the way to the solar surface. According to the studies [1-3], bodies with high core temperatures are abundant in astrophysical environments. And the high temperature differential between the core and surface of such bodies induce radiative and convective heat transfer. That is, energy transmission through the plasma of the solar interior

is mainly by means of radiation and convection [4]. This suggests that the dynamics of radiation and convection allow for the progressive reduction in the transmitted energy.

Heat transfer mechanisms are of high significance in electrically conducting and magnetized fluid flow problems commonly encountered in astrophysics and engineering. Several works have been done in the literature that borders on heat transfer problems. In their study on the flow of a two-component plasma model in a porous rotating sphere investigated the effect of radiation on the temperature distribution in the interior of astrophysical bodies [5]. Whereas as the study [6] investigated the flow of a low density thermally radiating two-component plasma in the presence of mass transfer and Hall current; while, the study [7] studied a two-component plasma model with

radiative heat transfer past a slowly rotating porous hot sphere under an optically thin gas approximation. Also, [8] studied the effect of slip parameter on hydromagnetic oscillatory flow combined with heat and mass transfer. Much work has equally been done in the literature that incorporated radiative and convective heat transfer [1, 5, 7, 9], as applicable in stellar interiors.

In the studies above the plasma density was considered constant; that is, the effect of plasma compressibility assumed negligible. But, the statistics presented by the researcher [10] shows that the plasma density within the solar sphere and atmosphere vary with radial distance from the core. The study [11] have also shown that ignoring the effect of compressibility is dangerous. This is because experiment indicates that free convection motion in a flow regime is usually caused by changes in the local density due to variations in the hydrostatic pressure resulting in isotropic acceleration of the fluid. The variability of solar plasma density was also acknowledged by the researches [12, 13] in their studies to investigate the effect of gravity on the stability of solar plasma and the temperature distribution in the solar sphere, respectively. In this study therefore, we considered an exponential representation of the solar plasma density.

## 2. The Physics of the Problem and the Governing Equations

The problem models the flow temperature of a magnetized plasma in a sphere, by considering an exponential varying density of the form;

$$\rho'(r) = \rho_0 e^{-\alpha_0 r} \quad (1)$$

and magnetic field of the form;

$$B = B_0 + B' \quad (2)$$

If the sphere is allowed to rotate slowly about the azimuthal with an angular velocity,  $\Omega_0$ ; its magnetic Reynolds number,  $R_m$  would be far less than 1 (i.e.  $R_m \ll 1$ ), such that the induced magnetic field  $B'$  can be neglected, leaving only the applied magnetic field  $B_0$  to influence the flow.

Therefore, if  $\rho'(r')$ , the plasma density as a function of radial distance,  $r'$ ;  $(u', v', w')$ , the velocity components in the orthogonal  $(r', \theta, \phi)$ -directions, respectively of the spherical coordinate system;  $p'$ , the pressure;  $T'$ , the temperature;  $q'_r$ , the radiative heat flux vector;  $\mu$ , the dynamical viscosity;  $\chi$ , the permeability;  $C_p$ , the specific heat capacity at constant pressure;  $\kappa$ , the thermal conductivity;  $\sigma$ , the Stefan-Boltzmann constant;  $\gamma$ , thermal expansivity;  $\epsilon_r$ , the absorption coefficient and  $T_\infty$ , the wall temperature of the medium at equilibrium with  $T_w$ , the wall temperature which was kept constant so that temperature differential is large enough to warrant radiation or radiative heat transfer, then the governing equations based on the Navier-Stokes equations under the Boussinesq approximation, following studies [1, 5, 7, 14, 15] are presented in dimensional forms as follows;

$$\nabla \cdot (\rho' V') = 0 \quad (3)$$

$$\rho' [(V' \cdot \nabla) V'] = -\nabla \cdot P' + \mu \nabla^2 V' - \frac{\mu}{\chi} V' - \sigma B_0^2 V' + \rho' \beta g (T' - T_w) \quad (4)$$

$$\rho' C_p [(V' \cdot \nabla) T'] = K \nabla^2 T' - \nabla \cdot q'_r \quad (5)$$

and

$$\nabla \cdot \nabla q'_r - 3 \sigma \epsilon_r^2 q'_r - 4 \sigma \epsilon_r T'^3 \nabla T' = 0 \quad (6)$$

where,  $\alpha_0$ , the density parameter; and  $\rho_0$ , dimensionless density;  $g$ , the accelerations due to gravity and the superscript (\*) indicates dimensional variable. Equations (3) to (6) are respectively the continuity, momentum, heat distribution and the generalized Rosseline radiative heat transfer flux.

As in the case [6, 12], the plasma gases in the intergalactic and interplanetary layers are seen to be rarefied. The plasma optical property,  $\epsilon_r$  in such case is far less than one (i.e.,  $\epsilon_r \ll 1$ ), so that the gas in these region are mostly regarded as optically thin. Hence, the generalized Rosseline radiative heat flux integro-differential equation for the optically thin limit can be expressed as;

$$\nabla q'_r = 16 \epsilon_r \sigma (T'^4 - T_\infty^4) \quad (7)$$

Furthermore, from the statistical data presented by [12], as well as the models of [1, 15], the temperature difference between adjacent layers of the plasma is not much compared to each other, thus;

$$T' = T_\infty + \phi \quad (8)$$

where,  $\phi$  is a small temperature correction factor, such that,  $O(T') \gg \phi \gg O(T_\infty)$ , then Equation (7) reduced to;

$$\nabla q'_r = 16 \sigma \epsilon_r T_\infty^3 (T_\infty + \phi) \quad (9)$$

such that the heat transfer equation becomes;

$$\rho' C_p [(V' \cdot \nabla) \phi] = K \nabla^2 \phi - 16 \sigma \epsilon_r T_\infty^3 \phi \quad (10)$$

Subjecting Equations (3), (4) and (10) to the following boundary conditions;

$$T' = T_c; u', v', w' = 0; T' = T_w$$

then introducing the following non-dimensional relations;

$$r = \frac{r'}{R_0}, (u, v, w) = \frac{(u', v', w')}{\Omega_0 R_0}, \rho(r) = \frac{\rho'(r)}{\rho_0}, \Theta = \frac{\phi - T_\infty}{T_w - T_\infty}$$

$$Re = \frac{\rho_0 \Omega_0 R_0^2}{\mu}$$

the rotational Reynolds number,

$$\chi^2 = \frac{R_0^2}{k}$$

the porosity parameter,

$$M^2 = \frac{\sigma B_0^2 R_0^2}{\mu}$$

the magnetic parameter (the Hartmann number),

$$G_r = \frac{\rho_0 R_0}{\Omega_0 \mu} \gamma g (T' - T_\infty)$$

$$Re \rho \left[ u \frac{\partial u}{\partial r} + \frac{v}{r} \frac{\partial u}{\partial \theta} - \frac{v^2 + w^2}{r} \right] = \left[ \nabla^2 - \chi^2 - M^2 - \frac{1}{r^2 \sin^2 \theta} \right] u - \frac{2}{r^2} \frac{\partial v}{\partial \theta} - \frac{2v}{r^2} \cot \theta - \rho G_r \Theta \tag{12}$$

$$Re \rho \left[ \frac{\partial v}{\partial t} + u \frac{\partial v}{\partial r} + \frac{v}{r} \frac{\partial v}{\partial \theta} + \frac{uv}{r} - \frac{w^2}{r} \cot \theta \right] = \left[ \nabla^2 - \chi^2 - M^2 - \frac{1}{r^2 \sin^2 \theta} \right] v + \frac{2}{r^2} \frac{\partial u}{\partial \theta} + \rho G_r \Theta \tag{13}$$

$$Re \rho \left[ u \frac{\partial w}{\partial r} + \frac{v}{r} \frac{\partial u}{\partial \theta} + \frac{uw}{r} + \frac{vw}{r} \cot \theta \right] = \left[ \nabla^2 - \chi^2 - M^2 - \frac{1}{r^2 \sin^2 \theta} \right] w + \rho G_r \Theta \tag{14}$$

$$Re Pr \rho \left[ u \frac{\partial \Theta}{\partial r} + \frac{v}{r} \frac{\partial \Theta}{\partial \theta} \right] = (\nabla^2 - N^2) \Theta \tag{15}$$

### 3. Method of Solution

Finding analytical solutions for problems in the physical sciences and engineering has always been a difficult task for researchers. This is because most of the equations that arise from such problems are usually non-linear such that solutions

$$f(r, \theta) = f_0(r, \theta) + Re f_1(r, \theta) + Re^2 f_2(r, \theta) + Re^3 f_3(r, \theta) + \dots \tag{16}$$

However, for a sufficiently small  $Re$  (i.e.  $O(Re) \ll 1$ ), the higher-order terms of Equation (16) will make negligible additions to the solution. Therefore, ignoring the higher order terms, we truncate the perturbation series after first order correction term;

$$f(r, \theta) = f_0(r, \theta) + Re f_1(r, \theta) \tag{17}$$

The solutions to the resulting orders  $O(f_0)$  and  $O(f_1)$  equations for each of the flow variables are effected by using the following transformations;

$$\Theta_0(r, \theta) = \Theta_0(r) \sin \theta$$

$$w_0(r, \theta) = w_0(r) \sin \theta$$

$$u_0(r, \theta) = u_0(r) \cos \theta$$

$$v_0(r, \theta) = v_0(r) \sin \theta$$

$$\Theta_1(r, \theta) = \Theta_1(r) \sin 2\theta$$

$$w_1(r, \theta) = w_1(r) \sin 2\theta$$

the Grashof number,

$$Pr = \frac{\mu C_p}{\kappa}$$

the Prandtl number, and

$$N^2 = \frac{16 \alpha_r \sigma R_0^2 T_\infty}{\kappa}$$

the radiation or heat transfer parameter.

and eliminating the pressure gradients in the  $r$  and  $\theta$ -directions the governing equations in spherical coordinate system ( $r, \theta, \phi$ ) for the flow symmetrical about  $\phi$ -axis, become;

$$\rho \frac{\partial u}{\partial r} + \frac{2\rho u}{r} + u \frac{\partial \rho}{\partial r} + \frac{\rho v}{r} \cot \theta + \frac{\rho}{r} \frac{\partial v}{\partial \theta} = 0 \tag{11}$$

are not easy to obtain [16-19]. Generally, asymptotic solutions are sought for problems with such complex equations [19].

The non-dimensionalized equations above are highly non-linear and coupled, and as such are intractable. Therefore, we adopt the general perturbation method to obtain approximate solutions for the governing equations by setting up a perturbation power series about the Reynolds number,  $Re$ , since the sphere was allowed to rotate slowly. Thus, the solution to each of the flow variables is expressed in the form;

$$u_1(r, \theta) = u_1(r) (2 \cos^2 \theta - \sin^2 \theta)$$

and

$$v_1(r, \theta) = v_1(r) \sin 2\theta$$

and the results obtained are graphically presented using the Wolfram Mathematica computing software (Mathematic 9).

### 4. Results and Discussion

This section presents the results and discussion of findings. Figure 1, shows the temperature profile of the solar sphere, where the point 1.0 corresponds to the solar surface. While, Figures 2 and 3 show the temperature profiles for various values of radiation parameter,  $N^2$  and free convection parameter,  $Gr$ , respectively. For small values of radiation parameter,  $N^2$ , as indicated in Figure 2, increase in  $N^2$  leads to a reduction in temperature of the plasma.

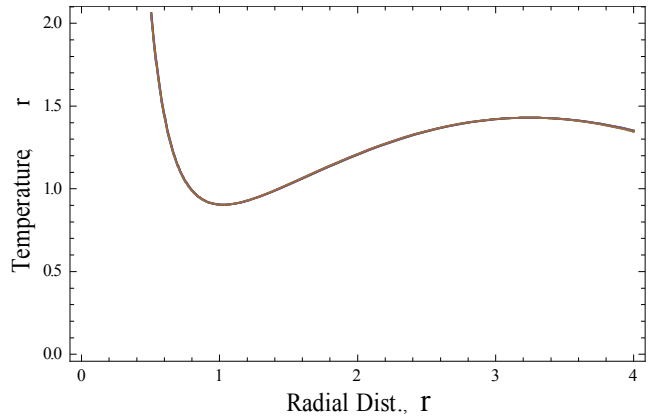


Figure 1. A Plot of Temperature,  $\theta(r)$  against Radial Distance,  $r$ : Showing Plasma Flow Temperature Profile.

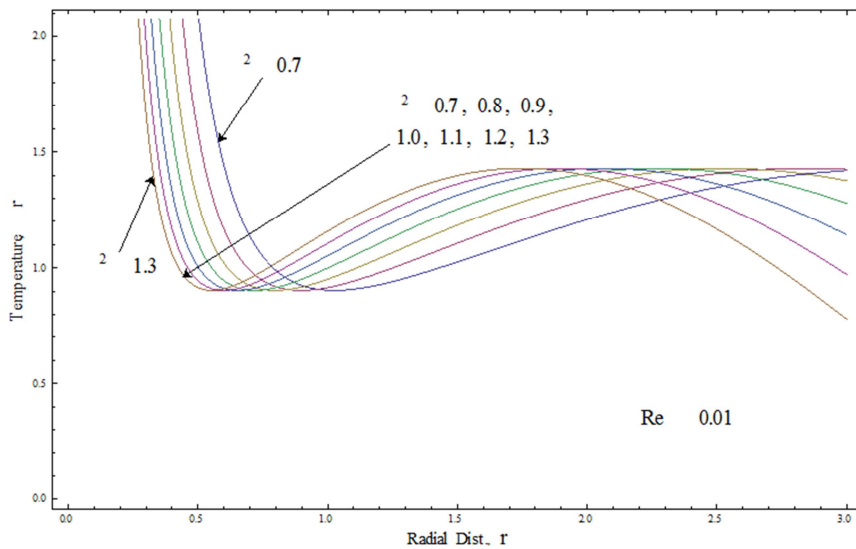


Figure 2. A Plot of Temperature,  $\theta(r)$  against Radial Distance,  $r$  with variation in Radiation Parameter,  $N^2$ .

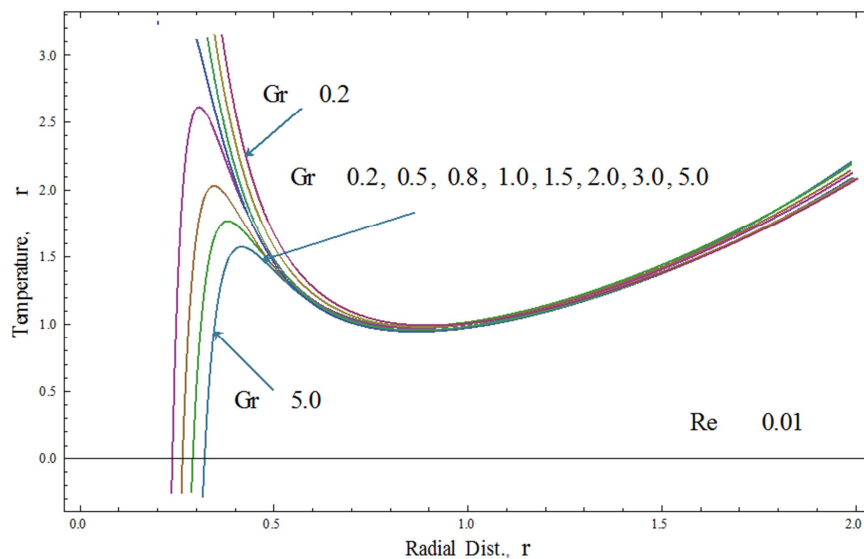


Figure 3. A Plot of Temperature,  $\theta(r)$  against Radial Distance,  $r$  with variation in free convection parameter,  $Gr$ .

The temperature reduction was as a result of loss of radiant energy emitted as gamma ray photons. However, just above the solar surface where the photons are released in the visible range as thermal radiation, the radiation parameter has a positive correlation with the plasma temperature. Similarly, the temperature decreases with increase in free convection parameter,  $Gr$ . Increase  $Gr$ , is seen to induce convective mixing of the plasma which leads to temperature reduction, though gradually near the solar surface.

Generally, the temperature profiles as established from the model was observed to decrease sharply from the core (at  $r = 0.25$ ) of the sphere to about,  $r = 0.7$  (i.e.,  $0.25 \leq r \leq 0.7$ ), but gradually from there to surface of the sphere, at  $r = 1.0$ . The region within  $0.25 \leq r \leq 0.7$  of the sphere corresponds to the radiation zone. Here high energy gamma ray photons are transmitted radiatively amidst scattering and absorption by plasma species. The drastic reduction in temperature was due to the interaction of the photons with the plasma species. While, within  $r = 0.7$  and  $r = 1.0$  corresponds to the convection zone. The high opacity in this region due to partially ionized heavy elements [20-22] renders radiation ineffective. Hence, the absorbed photons stir up the plasma to initiate convective motion. The convective mixing of the plasma accounts for the gradual decrease in temperature.

## 5. Conclusion

The thermal parameters of interest investigated in this study (that is, the radiation and free convection parameters) were seen to significantly influence the temperature distribution in the solar sphere. Both the radiation and free convection parameters are found to have negative correlation with the flow temperature within the sphere. That is, increasing the radiation parameter,  $N^2$  and free convection parameter,  $Gr$ , led to a decrease in the plasma flow temperature in the sphere. Whereas, above the sphere (i.e. at radial distances,  $r > 1.0$ ) where the plasma density is sparse, increasing the radiation and free convection parameters produced a corresponding increase in the plasma flow temperature. The decrease in the plasma flow temperature within the sphere with increase in the thermal parameters was observed to be more noticeable between radial distances,  $r = 0.25$  and  $r = 0.7$  (i.e.,  $0.25 \leq r \leq 0.7$ ; corresponding to the radiation zone of the solar interior) than between  $r = 0.7$  and  $r = 1.0$  (i.e.,  $0.7 \leq r \leq 1.0$ ; corresponding to the convection zone of the solar interior). This is attributable to the presence of partially ionized heavy elements in the convection zone which trap the high energy photons thereby reducing the rate of radiative heat loss.

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