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Applications of Math Microscope in the Event Horizon Telescope

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Abstract: The paper describes the insides - the basic concepts of the Math Microscope, demonstrates the results of Super-Resolution images obtained from the Event Horizon Telescope and analyzes the results of the movement of clusters of stars that go around the Black Hole. The presence of point objects - single stars in the SR image allowed us to implement a new breakthrough approach in the problem of SR images of Powehi Black Hole in the concept of MM. In the paper, we reviewed and illustrated new concepts: Invertability Indicators and Adequacy Characteristics of discrete Models of Apparatus Functions. With these new concepts, in the inverse problem, for the first time, we were able to answer simple questions: What are we dealing with? Moreover, have we solved the inverse problem? The paper demonstrates the "manual solution" of the problem of Reconstruction of AFs and Super-Resolution on MM. In the Discussion at the end of the paper, we pose the problem of creating two Artificial Intelligences for the automated solution of the R&SR problem with the interpretation of the SR results of BH images from EHT.

Keywords: Super-Resolution, Conditionality, Apodization in Inevitability, Modulation Transfer Function, Convolution, Fourier Transform

1. Introduction

Modern methods (such as regularization) for solving inverse problems are fundamentally ineffective, because they assume "a priori smoothness of solutions" [1]. We proceed from "a priori non-smoothness of solutions" - solutions consist of their points (in our case, these are clusters of stars). Point objects allow us to Reconstruct Apparatus Functions (AF) or Antenna Pattern (AP) of the Event Horizon Telescope (EHT).

The identification of points - clusters of stars with the estimation of the values of the achieved Super Resolutions is the basis of our methods.

We emphasize that the methods based on the Lagrangian formalism as in regularization have little to do with the methods of discrete mathematics with their own peculiarities.

Methods of Reconstruction and Super Resolution in Mathematical Microscope (R&SR in MM) of objects are intended for the Intelligent Analysis (IA) of data on objects observed through measuring devices whose AF or AP are not defined, are known with errors and even when they are precisely known. The methods of R&SR objects are based on the (Apodization search or) Reconstruction of mathematical models of discrete reversible AF O with minimal Nor(R), $R=O^{-1}$. Conditionality is the main setting of (AF) O to increase resolution as result of focusing MM. The conditionality is numerically equal to the reciprocal of the minimum value of Modulation Transfer Function (MTF) $|(M(O)|$ or the magnitude of this gap – $|(M(pO)|$. We introduce the magnitude SR of the estimating Super-Resolution [1-3]. The concept of the MM we applied for an image from the EHT.

2. Basic Concepts, Insides of the MM

2.1. Main problem, Reconstruction of AF

With the Conditioned SR (CSR), we will associate the problem of choosing a working discrete AF model O with an

invertible R=O^{-1} and a small inverse norm Nor(R)=‖R‖. If Nor(R) is large, then we are forced to reduce Nor(pR) by increasing the conditionality parameter DI (from word DIAPAZON) to obtain acceptable Nor(pR).

Note that Nor(pR) is the response to noise (or is there a standard deviation σ gain of white noise, Nor(pR)*σ), which determine the accuracy of solving the inversion problem in the presence of additive white noise σ.

The task of choosing the conditioned AF pO=pR^{-1} is posed as a minimum problem (maximum accuracy) [2, 3]:

$$\min_{LO}\{\|pR\| \mid Err(pO) \le err\}, \; LO = \{pO \mid [Loc, SDx, DI]\}, \quad (1)$$

Loc is the integer length of the domain of definition, taking into account the addition of one to zero. If the grid is fine, then Ste is an array of digitizing steps. The transition to an image on a coarse grid is obtained for ste> 1 from array Ste. In addition, if we have a regular pixel image, then we can interpolate the AF O and the image at a smaller step dx<1 from array Dx.
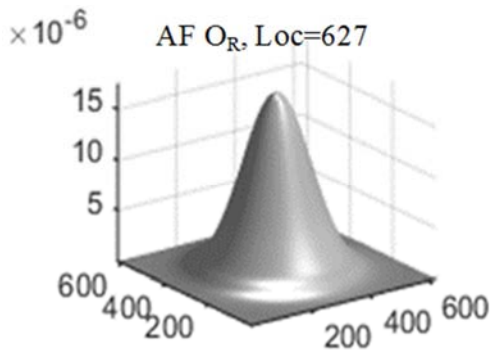
2.2. The Characteristics Adequacy of AF Model

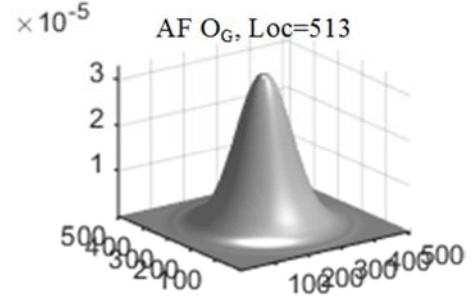The Characteristics Adequacy (CAM) of the discrete AF Model pO [2, 3] is:

$$CAM \; pO=TAB\{x=Nor(pR), \; y=Err(pO), \; z=II(pR*O)\}. \quad (2)$$
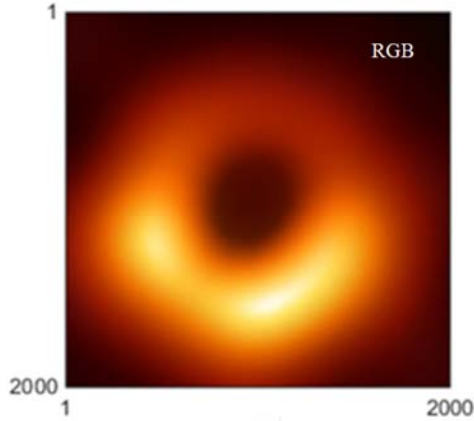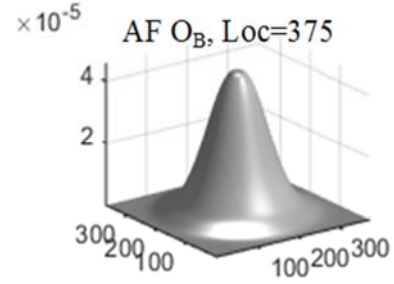

(a)

(b)

(c)

(d)

Figure 1. (a) there is a fragment of the BH image obtained with EHT on a full grid with ste=1, (b-d) there are reconstructed AF O$_R$, O$_G$ and O$_B$.

Iz=10^{-13} is the mantissa error (or Instrumental zero.) in calculating the Fourier coefficients. In connection with this Iz restriction, we consider the CAM AF zO variant, taking into account Iz the inversion of the two-dimensional MTF M(O) (in the record without indices):

$$M(zR) = \begin{cases} 1/M(O), \; for \; all \; |M(O)| > Iz \\ M(O) \end{cases} \quad (3)$$

$$CAM \; zO=TAB\{x=Nor(zR), \; y=Err(zO), \; z=II(zR*O)\} \quad (4)$$

Note that the most conditionality object (DI=1) in mathematics is the Delta Kronecker symbol DK=DK(i,j)={if i=0 and j=0, 1 else 0}, MTF M(DK)==1- identically equal to 1.

For ideal devices with AF O=DK, the formulation of problems of the type (1) "It is not necessary to construct MM to increase SR".
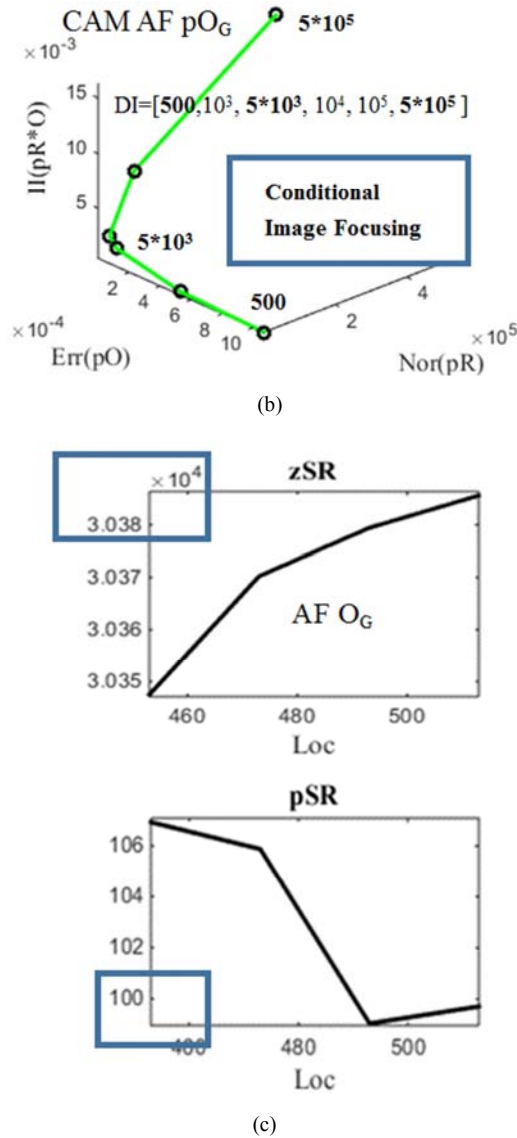

(a)

(b)

(c)
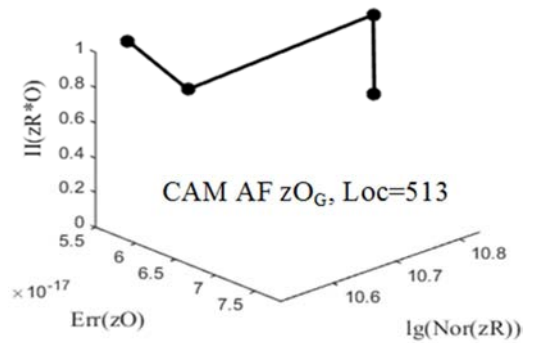
Figure 2. *(a) there is CAM AF zO$_G$, (b)- CAM AF pO$_G$, (c) – zSR values ~30000 are down by conditionality DI=5000 to pSR~100.*

2.3. Indicator of Invertability

In the method of regularization of Academician Tikhonov A. N. [1], the asymptotic statement connects the regularized approximate solution and the exact solution with the clause "if it (exact solution!) exists". We suggest using Indicator of Invertability (II) in addition of such statement.

Theorem [3, 4]: If the value of the II II(zR*O)=zR*O(0,0)=1, then there is an inversion zR=R=O^{-1}. If II(zR*O) <1, then we have incomplete Iz inversion of AF O to zR.

Note that if conditionality DI=(Iz)$^{-1}$=10^{13}, then in the reversible case II(zR*O)=1 CAM pO=CAM zO (2,4) are those same.

2.4. Super-Resolution Function, SR Values

If there is a normalization of AF O: $\sum O=1$, and at zero MTF M(O)(0,0)=1, then we will evaluate the super-resolution caused by the inversion of Iz by SR functions depending on

three parameters in LO (1):

$$pSR=\sum M(pR)*M(O)/\sum M(O),$$

$$zSR=\sum M(zR)*M(O)/\sum M(O) \qquad (5)$$

Note that SR≥1. If AF O=DK, then SR=1. In reversible cases, SR is a function with saturation by two parameters: the length of the domain of definition Loc and the conditionality DI, see Figure 2 (c).

Let us explain: The SR value is the ratio of the area of the "AF O spot" to the area of the smaller "AF pR*O spot" in terms of the MTFs (5) in the saturated case.

In addition, the maximal value SR is equal to zSR or simply is the "area of the AF O spot", measured in pixels (5)!

2.5. AF pO and zO Errors

The continuous parametric AF parO model, we associate the discrete AF O with the calculation (direct transformation) of the MTF M (O). If we build an inverse AF according to M(O), then we get the result of the AF zO with the Instrumental (mantissa) error Iz. The conditionality adjustment according to DI is that small values of M=M(O) move away from Iz by values 1/DI>min|M(O)| with the production of MTF pM. We realize the inverse transformation from MTF pM to AF pO [3]. Errors Err(pO), Err(zO) we estimated by the formulas [3]:
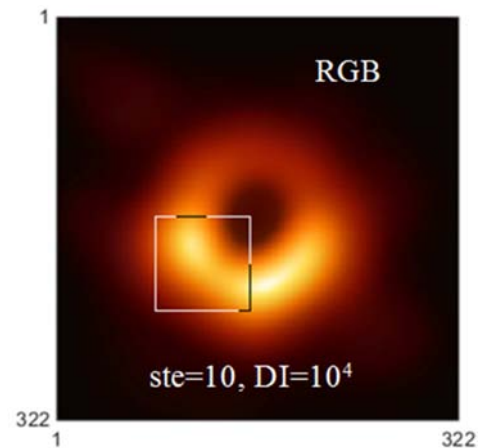
$$Err(pO)=SD(O-pO)/max(O),$$
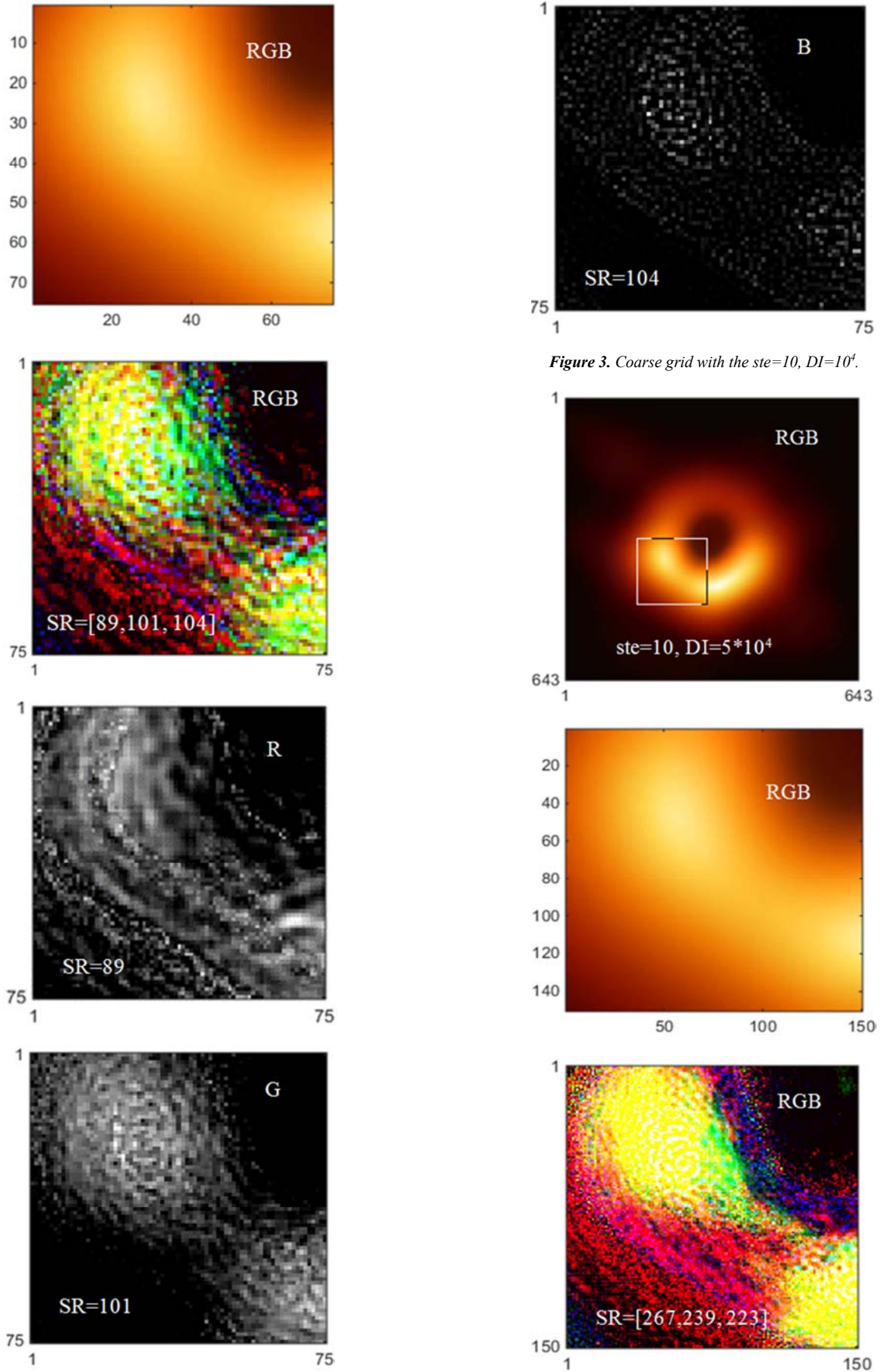
$$SD(O-pO)=sqrt(\sum(O-pO)^2/(Loc^2-1) \qquad (6)$$

and analogically Err(zO).

3. Illustrations of Basic Concepts, Insides of the MM

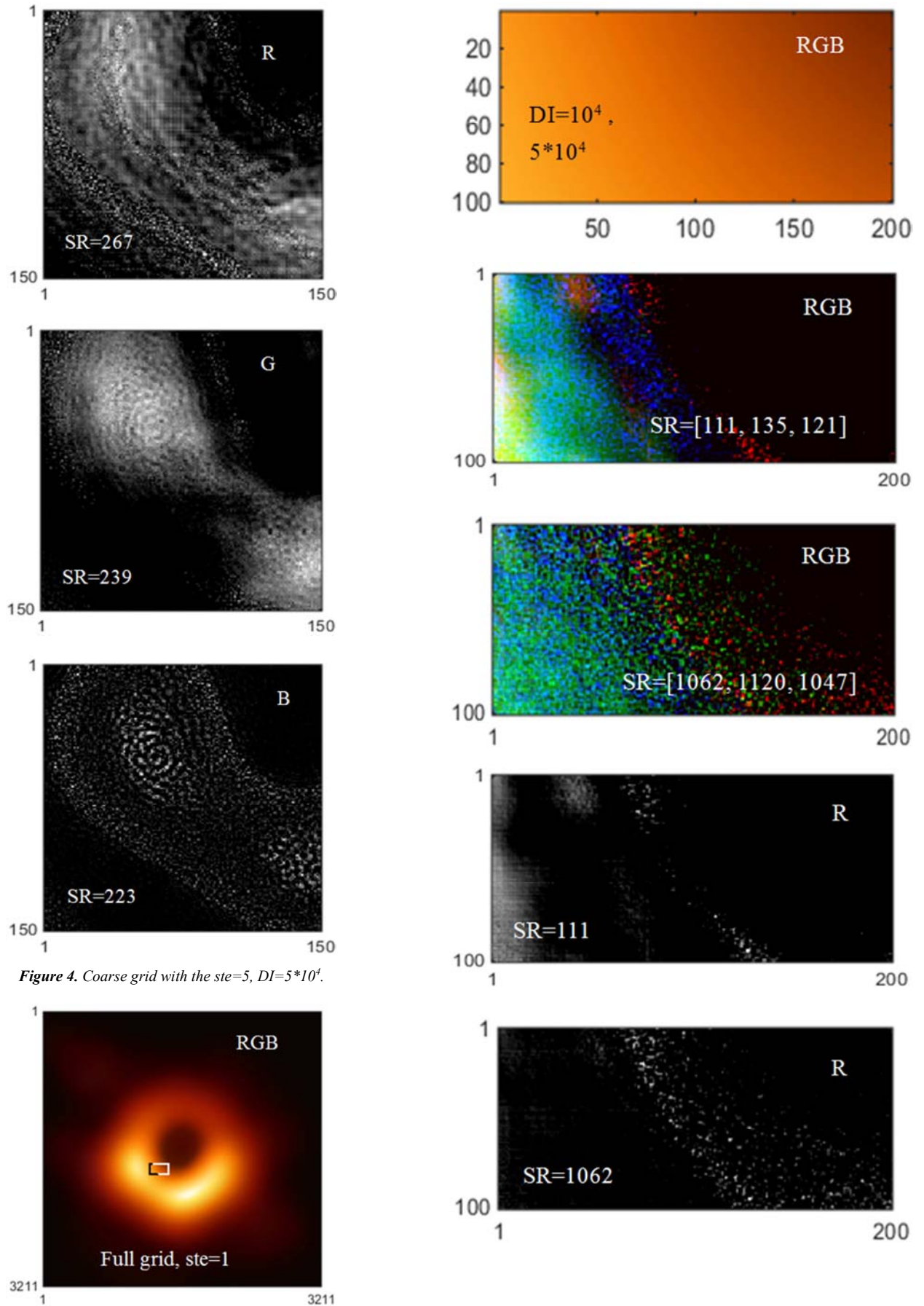### 3.1. Illustrations of Basic Concepts, Insides of the MM

The initial data are presented on coarse grids with the ste=10 and ste=5, SR is implemented with the conditionality DI=10^4 and 5*10^4.

Figure 3. *Coarse grid with the ste=10, DI=10⁴.*

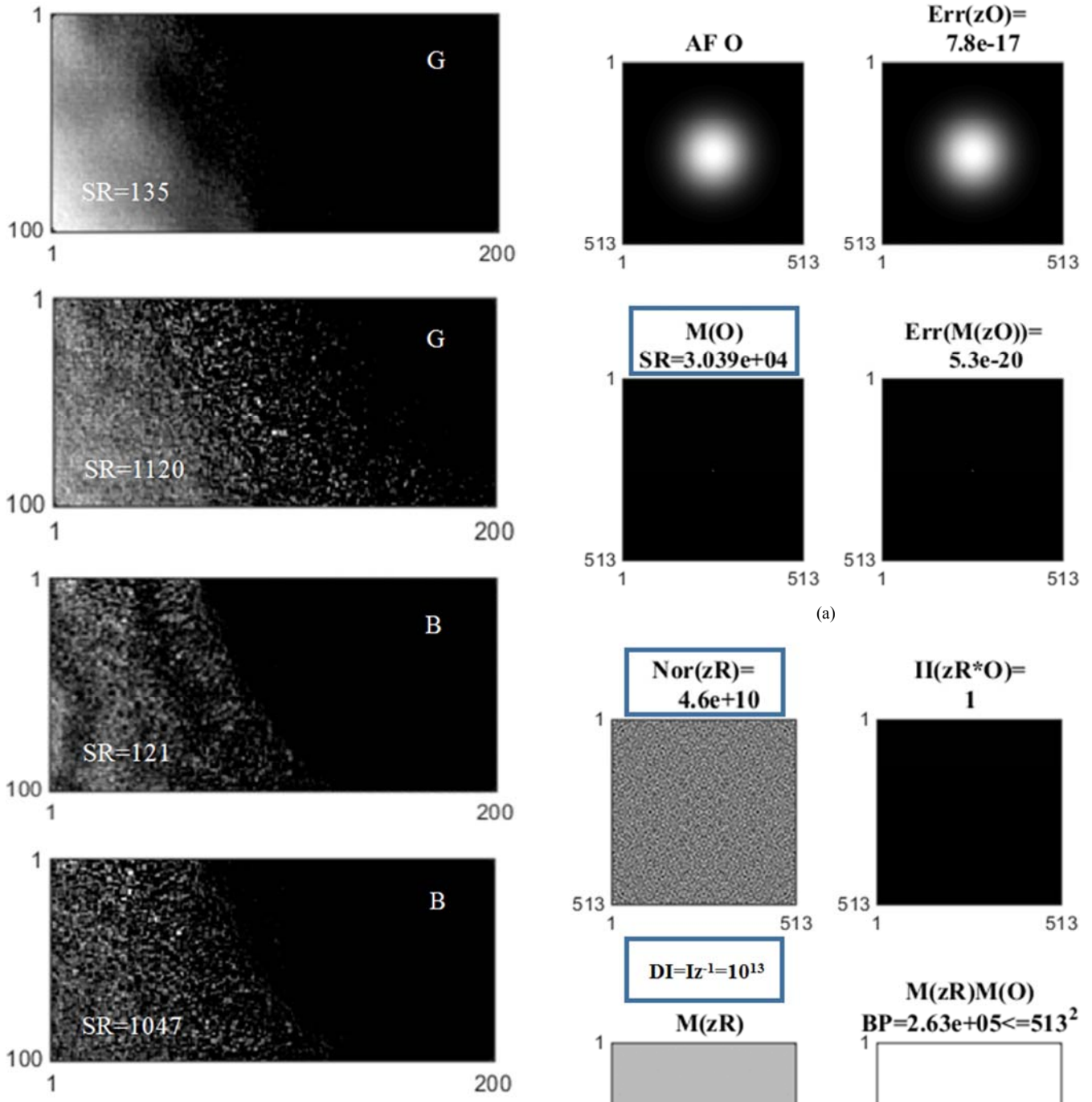**Figure 4.** *Coarse grid with the ste=5, DI=5*10⁴.*

Figure 5. *Full grid with the ste=1, with DI=10^4 and $5*10^4$.*

In the next three section, we demonstrate the insides of the MM on three conditionality DI different occasions.

3.2. Mantissa Errors, Maximum SR During Iz Inversion AF zO_G

According to the Theorem: II(zR*O)=1, the reversibility follows with the accuracy Iz:

zR=R=O^{-1}, zR*O=DK, MTF M(zR*O)==1, we have full Band Pass M(zR) M(O)==1, BP=5132 and we have the huge reaction to noise Nor(zR)~4.6*10^{10}.

Figure 6. *In (a-b) are Insides of Iz inversion (3-4), max SR~30000 during Iz inversion AF zO_G.*

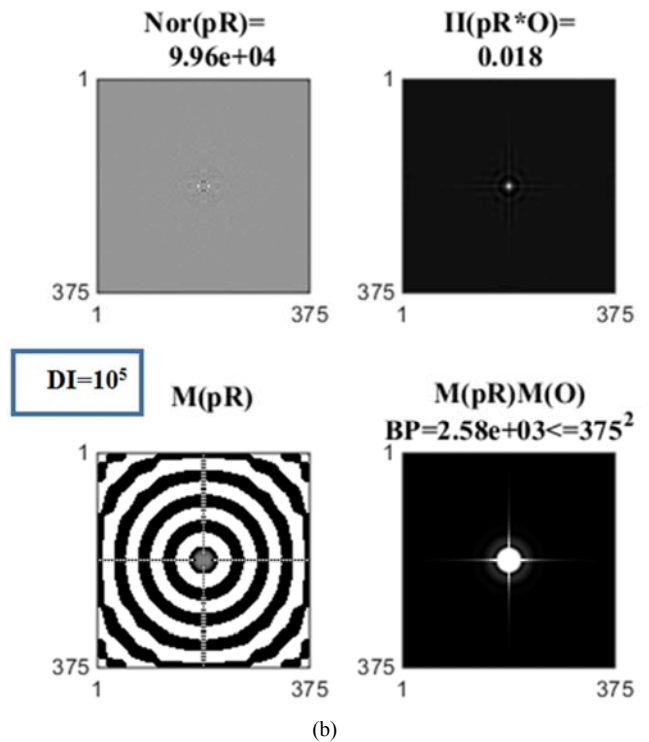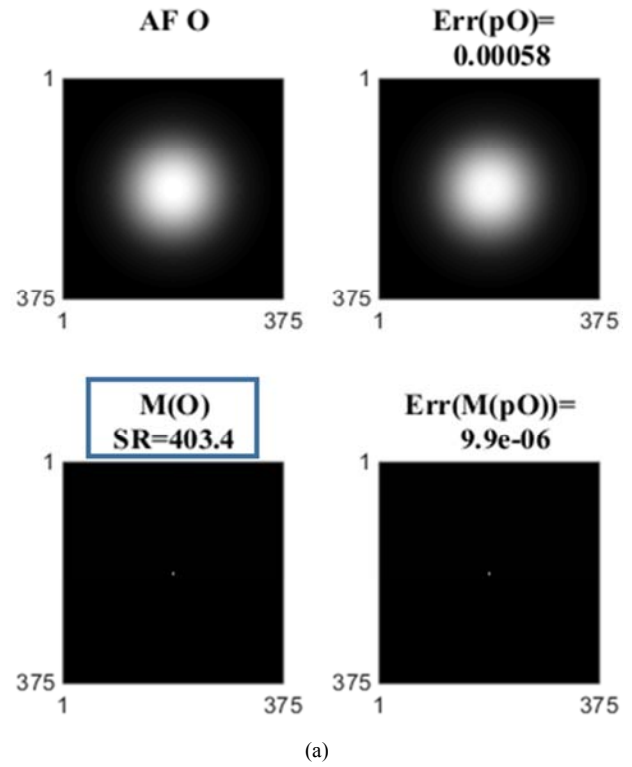### 3.3. Conditionality in Adjustment of Inversion AF pO_G, DI=2100

When setting the conditionality of the DI to 2100, we lose

in the reversibility of the II(pR*O)=0.0019, we get Err(pO)=2.8%, we reset the response to noise till DI~Nor(pR)=2100 and we get the small value SR~60 like in the theatrical binoculars.
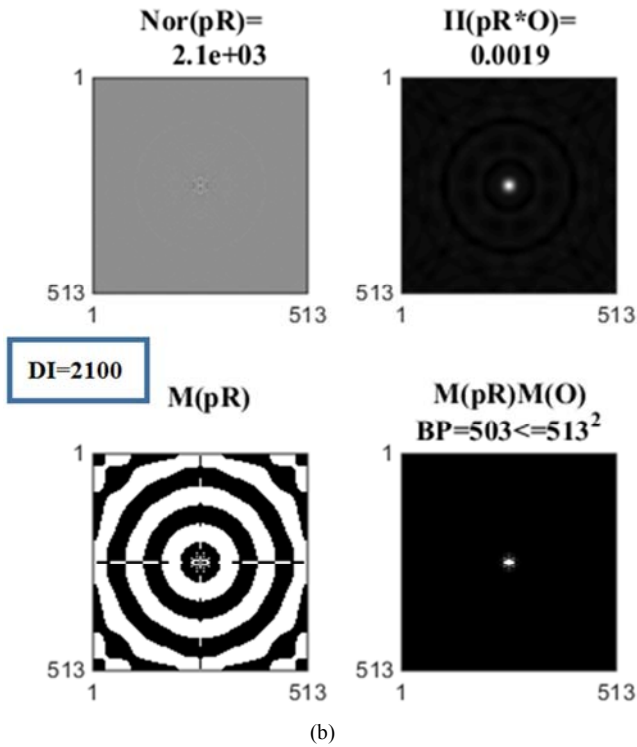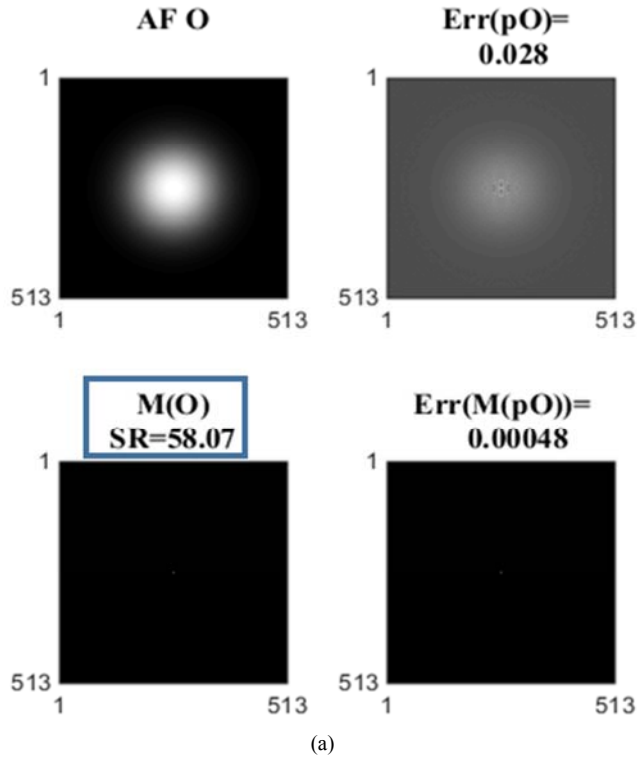
Figure 7. (a-b) are Insides of Conditional Inversion (1-2), conditionality in adjustment of inversion AF pO_G, DI=2100.

3.4. Conditionality in Adjustment of Inversion AF pO_B, $DI=10^5$

Figure 8. (a-b) are Insides of Conditional Inversion (1-2), Conditionality in adjustment of inversion AF pO_B, $DI=10^5$.

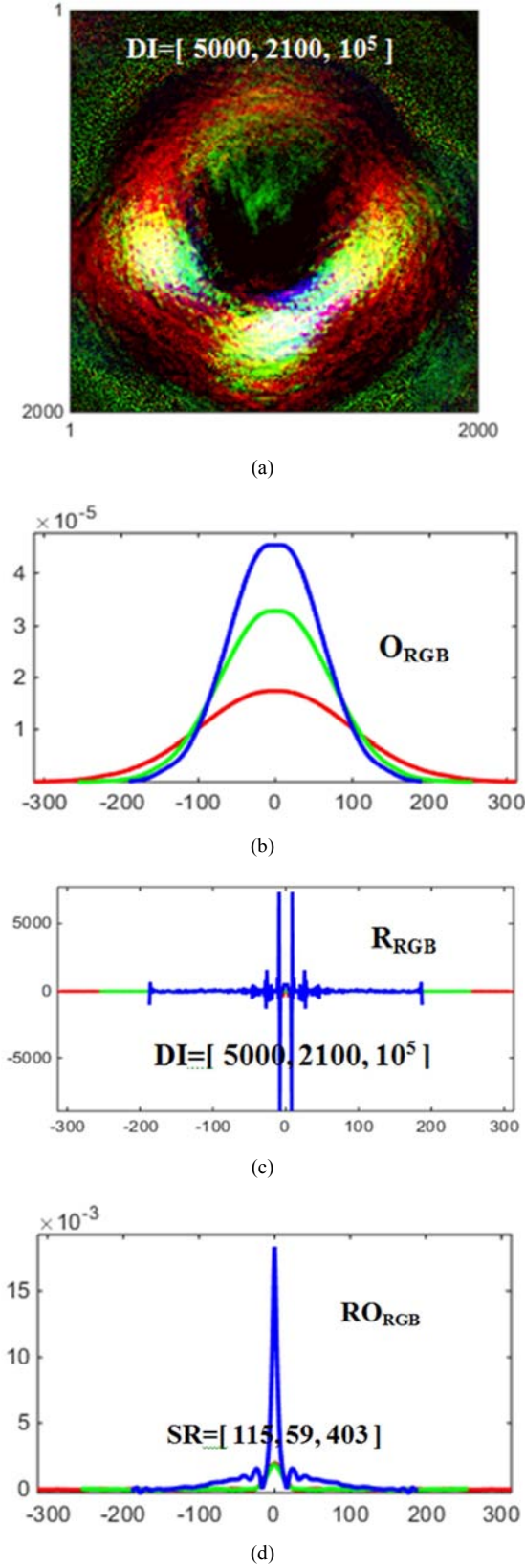When setting the conditionality of the DI to 10^5, we lose in the reversibility of the II(pR*O)=0.018, we get Err(pO)=0.028 we reset the response to noise till DI~Nor(pR)=10^5 and we get the good value SR~400 like in a children's microscope.

4. EHT Image Focusing in MM

(a)

(b)

(c)

(d)

Figure 9. *(a) - the CSR image, (b-d) – "cross sections" Reconstructed, Invers and Resolved AFs: $RO_{RGB}=\{R_R*O_R, R_G*O_G, R_B*O_B\}$.*
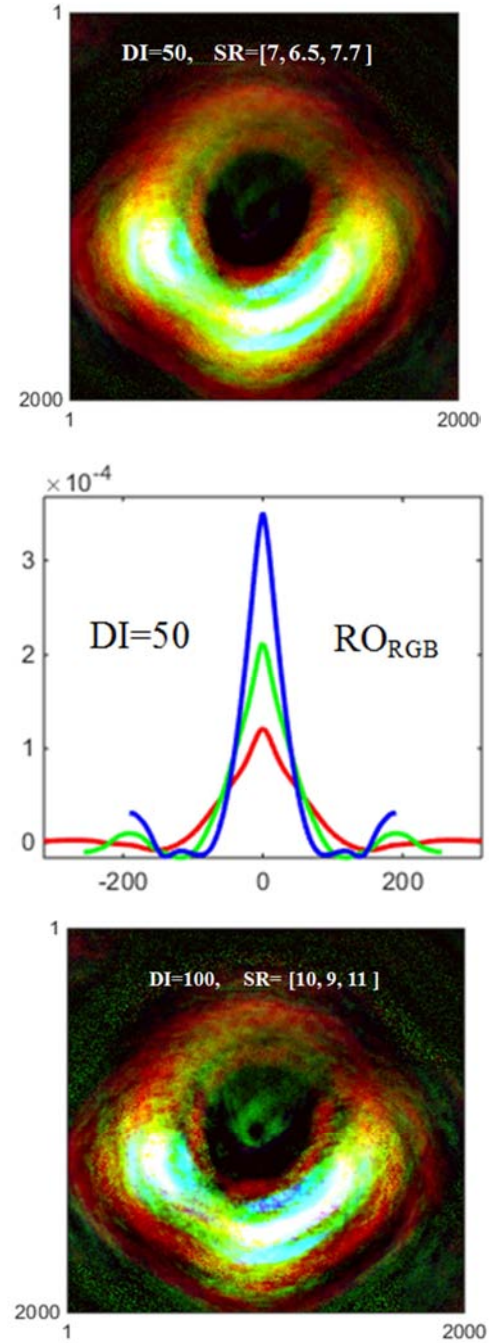
The Event Horizon Telescope (EHT) [9] consists of 8

receiving synchronized (in time by atomic clock) antennas in three radio bands {R, G, B}.

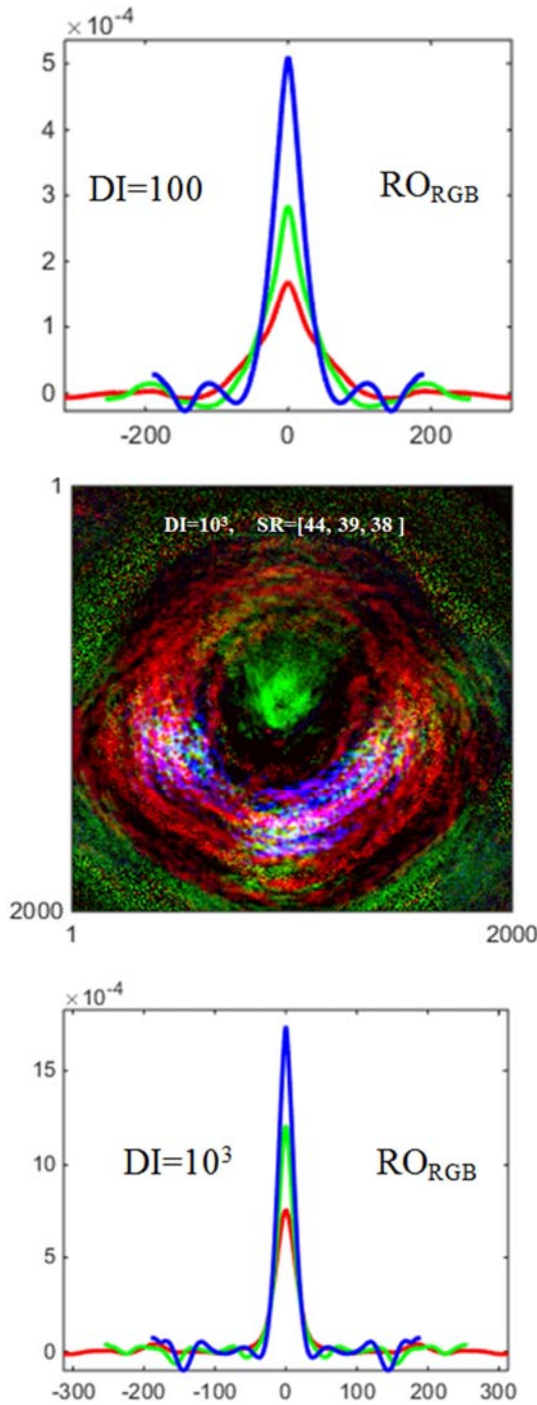Professor Katerine L. Bouman first implemented the mathematical docking [12] of data from 8 telescopes, turning them into one EHT with a large aperture (~ 10 000 km) and received the first image of the Powehi BH [2, 11], see in *Figure 1(a)*.

Focusing in MM is regulated by DI conditionality from the range of values [1, 1/min | M (O) |], that is, starting from the AF DK=DK^{-1} we pass pR=pO^{-1} to the reversed AF zR=zO^{-1}.

Figure 12 (b-c) shows the "cross-sections of the points" of which the images focused on conditionality at the output of the MM consist, see *Figure 11 (a), Figure 1(a)*.
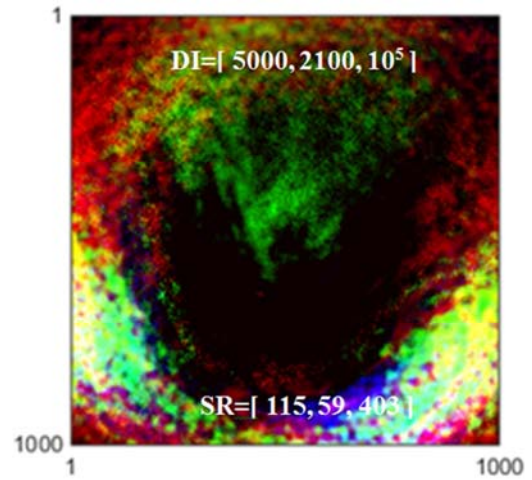
tomography, telescopes, in the manufacture of processors and memory, etc.

(a)

(b)

Figure 11. *(a) - CSR image with the highlighted central part and in the R channel (b).*
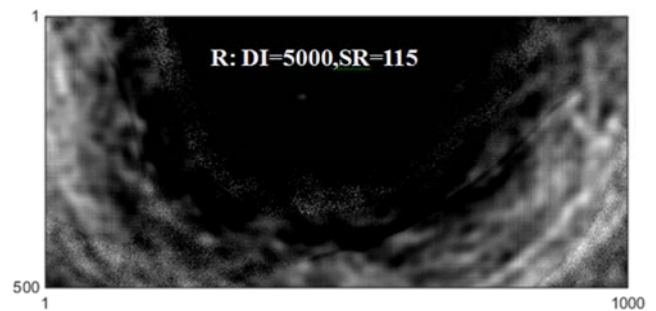
Figure 12. *CSR fragment of the image on channel R.*

Figure 10. *(a-c) – RO_{RGB}- resolved O_{RGB} AFs in three radio bands {R, G, B} for conditionality DI=50, 100 and 1000 with focused EHT images.*
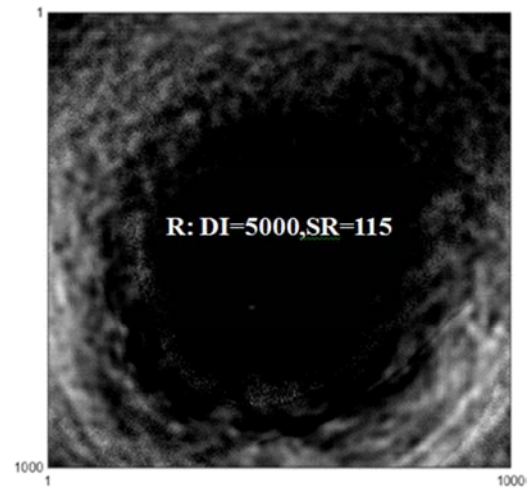
5. Analysis SR image from MM

In the CSR AF method, the resolved RO_{RGB} AFs are more "narrow-thin", compared to the original O_{RGB} AFs; the super-resolution values SR (5). The obtained CSR image correspond to a virtual EHT with an aperture from the Earth to the Moon.

Modifications of the CSR AF methods [2-8] can be implemented in electron microscopy, in new radar technologies, in synthetic aperture radars, CT, MRT

On channel R, we observe the movement of the BH shadow in the direction of the arrow, a cluster of stars in front of the BH shadow, two streamline stellar current tubes and clusters of stars behind the BH, see *Figure 13* below.
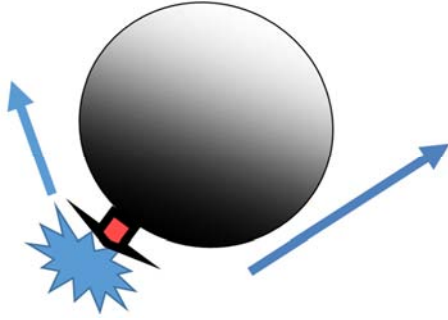
Figure 13. *The BH moves to a cluster of stars, the BH is traversed through two tubes of star streams.*

## 6. Results in Collaboration EHT

Below are the texts taken from the AIP messages, see [https://physicstoday.scitation.org/do/10.1063/PT.6.1.201904 11a/full/].

The imaging efforts came to a head at the weeklong June 2018 meeting in Cambridge, Massachusetts. Over the first few days, the four groups met separately but then began to share details about the intricacies of their analyses. Finally, on 25 June, the entire EHT imaging team viewed all four images. They weren't identical, but they all shared a fundamental feature: a roughly 40 μas photon ring surrounding an orb of darkness, the long-sought silhouette of a black hole. "It was a remarkable moment," says imaging team coleader Kazunori Akiyama of MIT
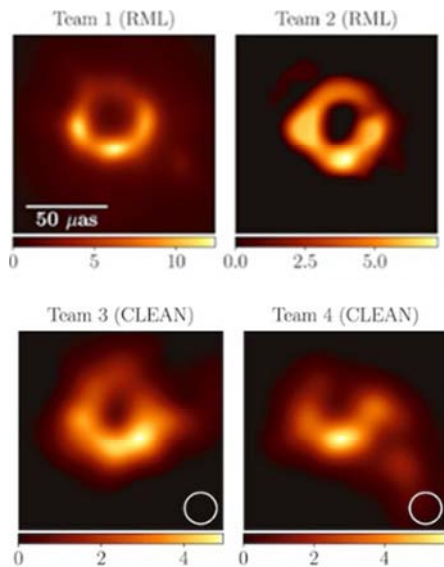
Figure 14. *Despite taking different approaches to analyzing the petabytes of EHT data, four teams ended up with relatively similar images of M87*.*

Not only did the images of the four groups agree with each other, but they also matched the output of models. Other members of the EHT team had developed General Relativistic Magneto Hydrodynamic Simulations (GRMHS) to first predict what EHT might see and then validate what it did see. The researchers ran thousands of simulations, each one with slightly different values for properties such as plasma temperature and the black hole's spin and magnetic flux.
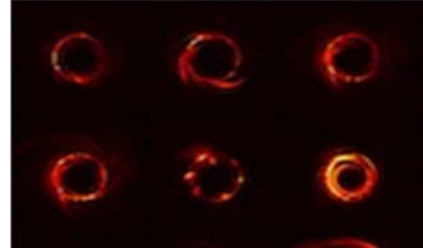

Figure 15. *In the array of GRMHS images, the modeled black holes look similar to the measured one.*

7. 2020 Nobel Laureates in Physics

Roger Penrose, Reinhard Genzel, and Andrea Ghez are to be awarded the 2020 Nobel Prize in Physics for their theoretical and observational work on black holes, see [https://www.nobelprize.org/prizes/physics/2020/summary/].

Genzel (Max Planck Institute for Extraterrestrial Physics and the University of California, Berkeley) and Ghez (UCLA) each led a team that advanced the techniques of speckle imaging and adaptive optics to obviate atmospheric turbulence and analyze the motion of stars tightly orbiting Sagittarius A*, the radio source at the Milky Way's center.
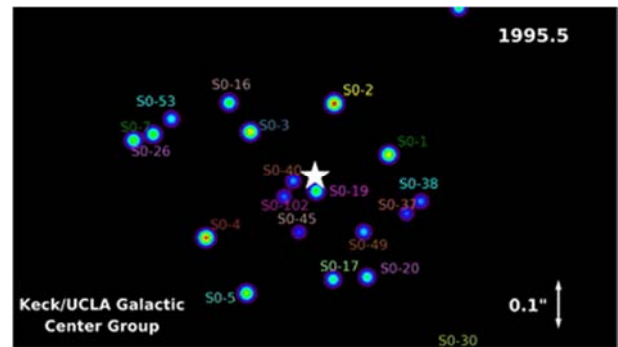
Figure 16. *The researchers concluded that only a black hole, weighing in at about 4 million solar masses, could be responsible for the orbits they observed.*

The decision of Swedish academics to award a prize for discoveries that are more theoretical in nature looks unusual.

8. Discussions, AI for MM

Our plans are to bring the SR value up to 2000-3000 times (like in a good optical microscope) on RGB channels and calculate all four frames of the surveys for April 5, 6 and 10, 11 April 2017, with control over the displacements of star clusters and, possibly, individual stars, as in *Figures 15*. Note that one eso frame for April 10 "weighs" 183.4 MB [2].

The results in this article were obtained by calculations on a laptop with a first generation Core i7 processor six years ago. One calculation option took approximately 3.5 minutes. Enumeration of a huge number of options is required on a powerful computer with an accelerator.

Of course, AF O_{RGB} reconstructions start with coarse grids like ste=10, 5 [9, 10]. The most valuable thing in this work is the AF O_{RGB} reconstructions (see *Figures 3-5* and only then

the SR images themselves (*in Figures 10, 13, 14*), in which point objects - stars are present.

Collaboration EHT is currently trying to verify the received data. In the first unsuccessful attempts to increase the resolution, see the six open access articles in [https://iopscience.iop.org/issue/2041-8205/875/1], point objects - star clusters were not identified. Even the original issue of AF or EHT Antenna Pattern estimation is not addressed.

Note that in the schematic drawing (*Figure 14*) BH "enters" the stellar matter, there is no vortex (as in *Figure 15*), we see reactive spitting in the G channel, see *Figure 9(a)*.

Note that based on SR images, the number of GRMHS; see *Figure 15* will be significantly reduced down to one model, as it should be in a physical experiment.

At various scientific sites in Russia with the presence of foreign scientists, a wide range of problems of the creation and use of Artificial Intelligence (AI) are currently being discussed.

Below we give the headlines of discussions about AI close to this work.

AI in Math Physics Modeling for Controlling Local Phenomena and Objects, two Projects:

P1: AI for Mathematical Microscope

P2: AI in Local Phenomena and Object Recognition

P1: AI for MM is intended for the Intelligent Analysis (IA) of data on objects observed through measuring devices whose Apparatus Functions (AF) are not defined, are known with errors and even when they are precisely known.

The estimation-reconstruction of the Reversible Apparatus Function $R=A^{-1}$ is realized by point objects in the Super-Resolved images by a human operator after viewing the SR images "manually". We propose to create AI for the automatic solution of the SR problem.

P2: AI in OR is intended for the analysis of parameters of local phenomena and objects for their recognition in images by using methods of Gradient Morphology (GM) [13, 14].

GM methods are designed to accurately determine the parameters of such objects as vortices in space images or in the BH images, Traffic Signs, Subtle Features of the Face, etc. Recognition is realized by the values of the parameters. We associate such complex programs with AI development.

We believe that the indicated problems P1 and P2 on the creation of AI in the next 2-3 years will be implemented.

Acknowledgements

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